VITRUVIAN MAN
BAROQUE DANCE

Is there a basic mathematical principle underlying Baroque dance and its "movements of the soul"

The Ancients, having taken into consideration the rigorous construction of the human body, elaborated all their works, as especially their holy temples, according to these proportions; for they found here the two principal figures without which no project is possible: the perfection of the circle, the principle of all regular bodies, and the equilateral square.—from *De divina proportione* by Luca Pacioli, illustrated by Leonardo da Vinci

A S A DANCER, I have experienced the art of dance to be a science of proportions, angles, and geometry. Thoughts, emotions and psychological states are expressed by angles within the body as well as lines in space defined by the moving body or bodies of the dancers.

When teaching historical dance, I begin by asking the dancers to envision Leonardo da Vinci's *Vitruvian Man* and to imitate the figure in the circle and then the figure in the square.
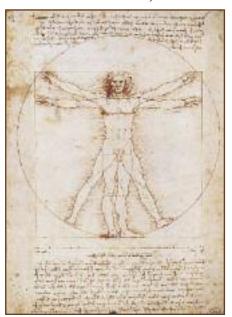
Leonardo used both image and text to express the ideas and theories of Vitruvius, a 1st-century Roman architect and the author of *De Architectura*. He related Vitruvius's writings on architecture to his own ideas on the structure of the human body.

If you open your legs so much as to decrease your height 1/14 and spread and raise your arms till your middle fingers touch the level of the top of your head you must know that the centre of the outspread limbs will be in the navel and the space between the legs will be an equilateral triangle. The length of a man's outspread arms is equal to his height.—from *The Notebooks of Leonardo DaVinci*, Vol. 1 (Dover, 1970)

The center of Leonardo's square is the pelvis. The center of his circle is the navel. According to Pythagorean tradition, a circle represents the spiritual realm and a square represents material existence. Thus, according to Leonardo's formulation, the human body represented the perfect marriage of spirit and matter. Leonardo related man to nature and considered the *Vitruvian Man* a *cosmografia del minor mondo* (cosmography of the microcosm). He believed the workings of the human body to be analogous to the workings of the universe. Bones were akin to the rocks of earth. Flesh was like the dirt or earth covering the rocky support. Blood moved like the water in a river running to the sea; it ebbed and flowed in a motion directed by the ocean tides.

I have found Leonardo's thoughts on architecture and the human body to parallel my thinking on dance choreography, and I often ask dancers to think of architecture as they carve

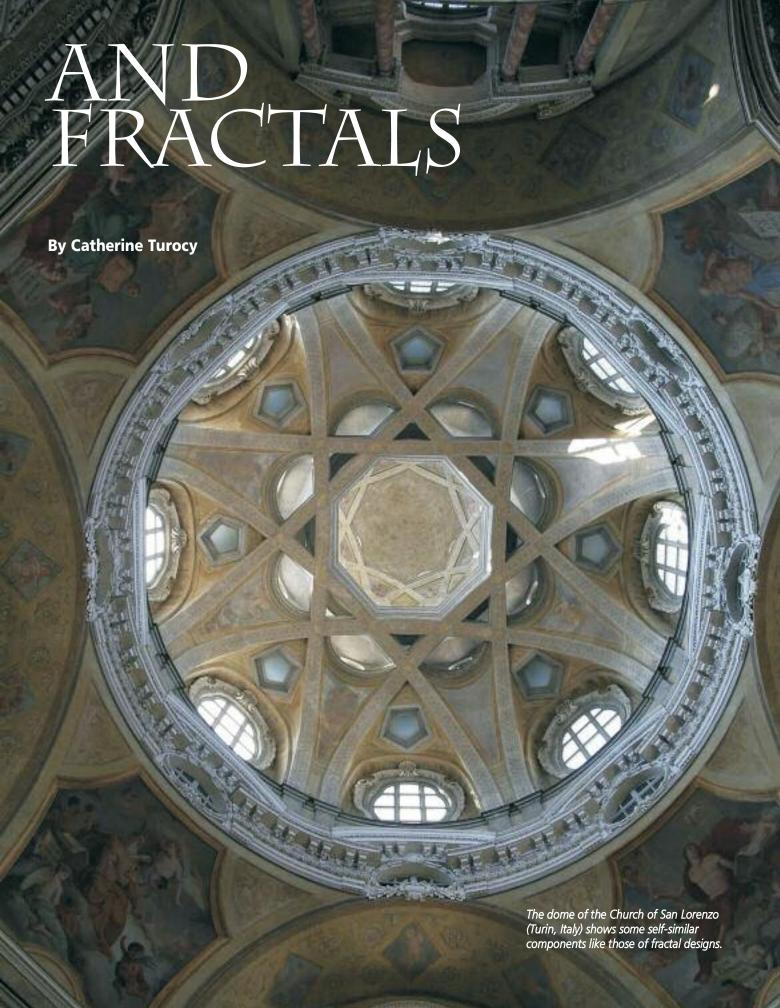

through space with their body.

This interest ultimately led me to "Fractal Components in the Gothic and in the Baroque Architecture," an interesting article posted on the Internet by Nicoletta Sala of the Academy of Architecture of Mendrisio at the University of Lugano. It takes a look at the mathematical proportions of Baroque architecture in relation to fractal principles.

Sala introduces the concept of self-similarity:

Another example of self-similarity in the Baroque architecture is in the dome of Church of San Lorenzo (Turin, 1666-1680), designed by the Italian architect Guarino Guarini (1624-1683). Guarini used the octagonal star to define the bearing structure of the dome. The self-similar components are an octagon and an octagonal star which are repeated in different scales.

Fractal geometry and its complexity can help to introduce the new paradigm in architecture…. This paper introduces only an approach to observe the Gothic and the Baroque architecture using a fractal point of view. The property of the self-similarity present in these two different styles has been chosen for an aesthetic sense; in fact the Gothic and the Baroque architects did not know the fractal geometry, because it is a recent discovery. Thus, it is possible to refer to the work as an "unintentional" use of the fractal geometry."

The concept of "unintentional" use of fractal geometry made me take a deeper look into my own experience in historical dance. I wondered if Leonardo da Vinci's writings may have the seeds of fractal theory embedded in the idea of the microcosm/macrocosm. Images of nature can remind today's dancer that, in the 15th century and well into the 18th centuries, art and science were

AND FRACTALS

By Catherine Turocy

The dome of the Church of San Lorenzo (Turin, Italy) shows some self-similar components like those of fractal designs.

united and both were concerned with the divine as found in nature. Truth and beauty existed in a work of art precisely because it was rooted in nature. Leonardo's drawing and text for the *Vitruvian Man* gives dancers a visceral sense of their own bodies as being tied to the universe and to the divine order of God.

However, it is not only the dancing body that reflects the cosmos, but also the choreography devised by the dancing master that reflects the carefully measured spatial patterns in proportion to the music.

Jennifer Neville, in her book, *The Eloquent Body* (Indiana University Press, 2004) explores the dancing masters of 15th-century Italy in relation to humanist ideas of divine proportion. She quotes Renaissance dancing masters referring to the *cosmografia del minor mundo* and talking about the "movements of the soul." In Giovanni-Andrea Gallini's book, *A Treatise on The Art of Dancing* (London, 1762), I have also found references to "the movement of the soul," which I believe to be a short code for the concept of *cosmografia del minor mondo*. The Enlightenment certainly challenged beliefs of man in relation to the divine, but were humanist concepts so integrated into the structure of dance that they remained covertly?

Neville outlines the efforts of Renaissance dancing masters to align dance with music and Pythagorean theory. Domenico da Piacenza (*c.*1400-*c.*1470) "…devotes a substantial part of the theoretical section of his treatise to explaining how dance shared the numerical basis of music and the other mathematical arts of the *quadrivium*." The *trivium* and the *quadrivium* comprised the seven liberal arts of the Latin school; the *trivium* consisting of grammar, logic, and rhetoric and the *quadrivium* of arithmetic, geometry, music, and astronomy. Dance was seen as a sister art to music. For myself, I often see the dancer's body as a musical instrument with its own voice. At times it blends with the rhythm and harmony and at other times it is a solo line. I experience sound visually, as a dancing body that senses proportions, colors and textures in physical planes.

Neville goes on to explain:

The art of dance was an aural as well as visual representation of the ratios between numbers and is made clear by Domenico in the theoretical section of his treatise. …one way in which Domenico represented the cosmos in his choreographies was by using the Pythagorean ratios to describe the temporal relationship between the four *misura*…it was also represented in the choreographic patterns….

The choreographic patterns in the Renaissance and the Baroque play with symmetry, repetition, and proportion. As an historical choreographer, I have been fascinated with these patterns. As someone who has danced many of the period dance notations published in the 18th century, I sometimes feel that the larger geometry of the paths traced in space by the dancing body are similar to the energy paths going through the body as it moves on a micro level. Am I experiencing a fractal model as I dance?

Johannes Kepler in his 1611 essay, *On the Six Cornered Snowflake,* mentions the "Divine Proportion" and the "Fibonacci Sequence":

It is in the likeness of this *self-developing series* that the faculty of propagation is, in my opinion, manifest: and so in a flower the authentic flag of this faculty is shown, the pentagon.

Kepler's idea of the *self-developing series*, together with Gottfried Leibniz's theory of recursions, gave a hint of the future concept of fractals.

The essence of fractals

For me as a dancer, it is the *self-developing series* that is the essence of fractals. I first imagine the shape of the movement in the dance in my mind, and it quickly passes through the nerves and muscles to manifest itself in the motion of my body, reflecting the choreographic structure of a published historical dance following the rules of *Divine Proportion*. Not all choreographies are based on such geometrical patterns, but quadrilles published in the 18th century certainly feel tied to geometrical perfection. I hope a dancer/mathematician will investigate the country dances and quadrilles of the 17th and 18th centuries from the perspective of fractals. Other disciplines like architecture are already holding conferences on Baroque architecture and

fractals. Why not dance?

The word "fractal" was first used by Benoit Mandelbrot in 1975 when he published *The Fractal Geometry of Nature*. His explanation of Chaos Theory is fascinating, and I do not pretend to comprehend all the principles. However, his discussion of biological systems, such as the branching of the circulatory and bronchial systems, seen as fractal models, makes sense. In Renaissance Man's vision of the tangible world, the Divine was absolute and resided in all things. God existed in every fragment of matter. The universe, its structure based on numbers and spheres, was a macrocosm existing in harmony with that microcosm of divine proportions. Could this concept of fractal geometry and the *cosmografia* as described by Leonardo be related?
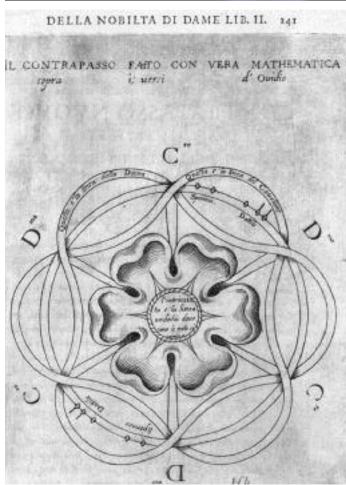
The fractal concept has many aspects, but the general idea is that fractals are infinitely self-similar or iterated. They have progressively ever-finer recursive detail at increasing magnifications. When reading through a Feuillet dance notation, I often feel I may be experiencing "progressively ever-finer recursive detail." The spiral in my mind is referenced in the twisting of my torso as I spiral in space, reaching for my partner's right hand and together our bodies trace a spiral path that is a recursive figure drawing us back to the original spiral I envisioned in my mind.

The blue fractal image on this page reminds me of Fabritio Caroso's dance diagram, below, with its interweaving lines. The symmetrical placement of the four outer ovals and the horizontal/vertical axis crossing in the center of the shape calls to mind the square formation seen in the minuet of Mr. Holt shown on next page.
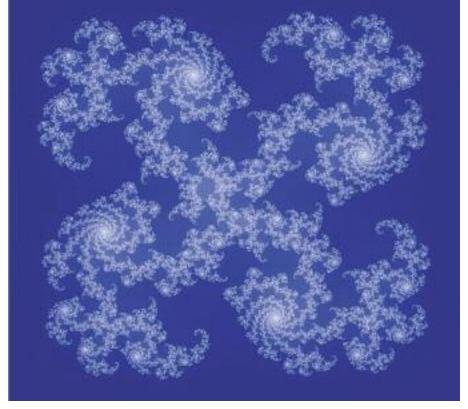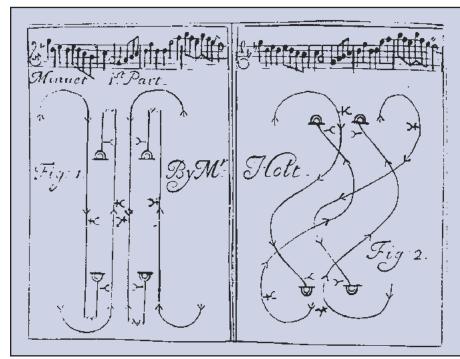
Julia set

In the context of complex dynamics, ...the Julia set and the Fatou set are two complementary sets defined from a function. Informally, the Fatou set of the function consists of values with the property that all nearby values behave similarly under repeated iteration of the function, and the Julia set consists of values such that an arbitrarily small perturbation can

Fabritio Caroso's Nobiltà di Dame, *printed in 1600, traces the paths of the dancers intertwining In space, anticipating the 20th mathematics behind the blue fractal design above.*

A minuet pattern by Mr. Holt, at left, published in 1711, anticipates a design by Daniel Renner based on a Julia Set, a formula devised by Gaston Julia, a French mathematician (1893-1933).

cause drastic changes in the sequence of iterated function values. Thus the behavior of the function on the Fatou set is "regular," while on the Julia set its behavior is "chaotic."(Wikipedia)"

With the help of the computer, graphic artist Daniel Renner has used formulas based on the Julia set to create designs that strike me as being very similar to Baroque choreography, such as the right hand figure of Mr. Holt's minuet published in 1711. This dance figure is common to many folk dances in various cultures and further embraces the concept of the circle within a square. Are the visual and kinesthetic geometric figures common to historical dance and fractal models a coincidence, or is there an underlying parallel joining art and the science of mathematics? Like Renaissance dancing masters using concepts of the divine proportion in their choreography, might our choreographers employ fractals in creating choreography today?

Another interesting fractal pattern is the nautilus. The spiral in the nautilus fractal reminds me of the minuet shown in Kellom Tomlinson's minuet figures (London, 1735). These are self-referencing in their patterns, with repetition in figures 2 and 5 and again in figures 3 and 4. In fact, the spiral, in some form, is found in all six figures.

Dance experiments

Although no formal studies of historical dance and fractals have been done thus far, other dance forms are currently engaging in fractal analysis. In their article, "How Fractal is Dancing ?" in the May 2008 Internet journal *Chaos, Solitons and Fractals*, Melkon Tatlier and Rana Suvak described an experiment:
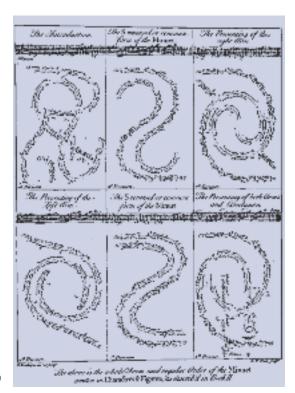
Fractal analysis was applied to the patterns formed on the dance floor by footwork while performing various dance figures. Several Latin-American dances were taken into consideration and the box-counting method was used to estimate the fractal dimensions of these patterns. Rumba was determined to produce the most fractal pattern with a fractal dimension of about 1.36 in the mesh size range used while Merengue exhibited the least fractal one with a dimension of about 1.16. The magnitude of the fractal dimension seemed to be mainly dependent on the simplicity/complexity of the dance figures as

well as the characteristic rhythm of the music dictating the basic footwork and figures performed. Well-known fractal shapes, such as the Koch curve might be used to provide insights for generating new dance figures."

In 2003, Wake Forest computer scienc faculty members Jennifer Berg and Tim Miller staged a production about computer science and dance called *Fibonacci and Phi* . As they explained:

> *Fibonacci and Phi* was collaboration among computer scientists, dancers, and musicians exploring mathematical concepts that have aesthetic appeal, capturing the ways in which mathematical beauty gives shape to nature and art and expressing the human response to these forms. Technically, the production tested the limits of parallel cluster computation in real-time multimedia and performance art. The result was a dance performance that wove together science and art in a way intended to draw new audience members into both realms.

A spiral nautilus design mimics the dance shown in Kellom Tomlinson's minuet figures (London, 1735).

FRACTALS IN DANCE

The Fibonacci sequence and the Golden Ratio, Phi, have been well-known to scientists and mathematicians since ancient times. While the Fibonacci sequence is deceivingly simple, this sequence is found in an amazing number of natural forms, ranging from the spiral of nautilus sea shells to the growth of leaves on a branch and seeds on the face of a sunflower. The sequence also is evident in the mathematical properties of fractals. Where the Fibonacci sequence appears, we generally find Phi, the Golden Ratio, as well. Phi's definition is also simple. If a line segment is divided into a smaller part A and a larger part B, and the ratio of the length of A to the length of B is the same as the ratio of the length of B to the whole segment, then this ratio is the Golden Ratio, Phi. Given its first explicit definition by Euclid more than 2,000 years ago, Phi has been associated with the discovery of irrational numbers, the proportions of classic paintings and architecture, and the geometry of beautiful phenomena of nature."

Final thoughts

Where to go with these thoughts? I would like to see further analysis by mathematicians in order to establish a firmer base of study between dance and fractals—a more in-depth scientific look at the relationship between fractal models and dance forms. I believe historical dance, with its roots in the *Cosmografia del minor mundo,* is particularly suited to such a study. Indeed, I hope the results of such research could invigorate today's performance of period dance as well as dance in general. It would be fun to see, if by using fractal models, today's choreographers could devise ballets based on the beauty and emotion found in a Julia set.

Dancer and choreographer Catherine Turocy is the co-founder of the New York Baroque Dance Company. This article is adapted from a talk given in September at the the Mark Morris Dance Center in New York City called: "What do Fractals in Nature, the Vitruvian Man, and Baroque Dance Have in Common?"